Topological Complexity

Abstract

For a path connected topological space X, the motion planning problem consists of constructing a program or a device, which takes pairs of configurations $(A, B) \in X \times X$ as an input and produces as an output a continuous path in X, which starts at A and ends at B. In this talk, we study a notion of topological complexity $\mathrm{TC}(X)$ for the motion planning problem. $\mathrm{TC}(X)$ is a number which measures discontinuity of the process of motion planning in the configuration space X. More precisely, $\mathrm{TC}(X)$ is the minimal number n such that there are n different "motion planning rules," each defined on an open subset of $X \times X$, so that each rule is continuous in the source and target configurations. We use methods of algebraic topology (the Lusternik–Schnirelman theory) to study the topological complexity $\mathrm{TC}(X)$. We give an upper bound for $\mathrm{TC}(X)$ (in terms of the dimension of the configuration space X) and also a lower bound (in terms of the structure of the cohomology algebra of X). We explicitly compute the topological complexity of motion planning for $\mathbb{S}^n, \mathbb{R}P^n, \mathbb{C}P^n$.